Key points of the last two lectures!

A lot of technical proofs this week. Here's the tl;dr summary:

- $\cos\theta = \frac{\langle \vec{v}, \vec{w} \rangle}{\|\vec{v}\| \|\vec{w}\|}$,
- $\vec{v}$ and $\vec{w}$ are orthogonal if and only if $\langle \vec{v}, \vec{w} \rangle = 0$,
- the projection of $\vec{v}$ on $\vec{w}$ is $p_{\vec{w}}(\vec{v}) = \frac{\langle \vec{v}, \vec{w} \rangle}{\langle \vec{w}, \vec{w} \rangle} \vec{w}$,

Projections

This vector describes how much $\vec{v}$ is in the direction of $\vec{w}$, and is known as a projection of $\vec{v}$ onto $\vec{w}$.

Definition

Let $\vec{v}$, $\vec{w} \in \mathbb{R}^n$ be non-zero vectors. Then the *projection of* $\vec{v}$ *onto* $\vec{w}$ is

$$p_{\vec{w}}(\vec{v}) := \|\vec{v}\| \cos\theta \hat{w}.$$

Can we write it using the scalar product?

Projections

Proposition

Let $\vec{v}$, $\vec{w} \in \mathbb{R}^n$ be non-zero vectors. Then the *projection of $\vec{v}$ onto $\vec{w}$* is

$$p_{\vec{w}}(\vec{v}) = \frac{\langle \vec{v}, \vec{w} \rangle}{\langle \vec{w}, \vec{w} \rangle} \vec{w}.$$

Proof.

Projections

Proposition

Let $\vec{v}$, $\vec{w} \in \mathbb{R}^n$ be non-zero vectors. Then the *projection of $\vec{v}$ onto $\vec{w}$* is

$$p_{\vec{w}}(\vec{v}) = \frac{\langle \vec{v}, \vec{w} \rangle}{\langle \vec{w}, \vec{w} \rangle} \vec{w}.$$

Definition

The *component of $\vec{v}$ on $\vec{w}$* is

$$component := |\langle \vec{v}, \hat{w} \rangle| = \|p_{\vec{w}}(\vec{v})\|.$$