FINAL EXAM

MATH 22 **NAME:** ________________________
Summer, 2024
Dr. Kejian Shi **ID #:** ________________________

SHOW YOUR WORK STEP BY STEP AND INDICATE THE ANSWERS CLEARLY.

1. Use a FULL **truth table** to determine whether the given **argument form** is **valid** or **invalid**. Write a sentence to explain how the truth table supports your answer. (24 points)

$$r \to t$$
$$\sim t$$
$$s \to t$$
$$\therefore \sim (r \wedge s)$$

2. Prove **directly** if the statements that are true, give **counterexamples** to **disprove** those that are false.

(a). "$\forall\, n \in \mathbb{Z}^+ - \{1\},\ p|n^2 \to\ p|n,\ if\ p\ is\ prime$." (16 points)

(b). "$\forall\, m, n \in \mathbb{Z}, 2m + 3n\ is\ odd\ \leftrightarrow\ (m\ is\ even\ \wedge\ n\ is\ odd)$." (10 points)

3. Prove **indirectly**.

"$\forall n \in \mathbb{Z}^+, (\ \forall k \in \mathbb{Z},\ n \neq k^2) \rightarrow \sqrt{n}\ \notin \mathbb{Q}$." (Hint: Part 2(a) might be useful.) (25 points)

4. Use **mathematical induction** to prove that $\forall\ n \in\ Z^+ \cup \{0\}, \prod_{i=0}^{n}(2i+1)! \geq \left((n+1)!\right)^{n+1}$. (25 points)

5. Given **sets** A, and B, use **set properties** to simplify $(A \cup B) - (B - A)$, indicate **every** property used at **each step**. (11 points)

6. For each of the cases, give an **example** of **function** $f: \mathbb{Z}^+ \to \mathbb{Z}^+$. Verify your examples. (24 points, 6 each)

(a). one-to-one and **onto** but NOT identity function

(b). one-to-one but NOT **onto**

(c). onto but NOT **one-to-one**

(d). NEITHER **one-to-one** NOR **onto**

7. Solve the following **counting** and **probability** problems. (15 points, 5 each)

(a). How many **permutations** of the letters in word INCOMPREHENSIBLE have three vowels in the first three positions and two consonants in the last two positions?

(b). A club consists of 6 freshmen, 7 sophomores, 8 juniors and 6 seniors. How many **committees** of 5 people from **at least** three classes?

(c). What is the **probability** that a five-card poker hand from a standard deck has **full house** with a **non-face card pair**?

8. Suppose $S \neq \Phi$, $S = \bigcup_{i=1}^{k} S_i$, and $S_i \cap S_j = \Phi$ if $i \neq j$. Define **relation** R on S by $xRy \Leftrightarrow x, y \in S_i$ for some $i \in \{1, 2, \cdots, k\}$.

(a). Prove that R is an **equivalence relation**. (18 points)

(b). Describe the **equivalence classes**. (4 points)

9. In each part, either **draw a graph** with the given specifications or **explain why** no such graph exists. (28 points, 7 each)
Remark: NO PARTIAL CREDIT FOR EACH PART.

(a). A **graph** with **4 vertices** that has both a **Hamiltonian circuit** and a **Euler circuit**.

(b). A **connected graph** with **5 vertices** that has circuits but neither a **Euler circuit** nor a **Hamiltonian circuit**.

(c). A **connected** and **circuit-free** graph with **7 vertices** and **total degree** 12.

(d). A **binary tree** to represent expression $\big(a/b - c \cdot (d + e)\big) - (f + g/(h - i))$.