[redacted]

**<u>SHOW YOUR WORK STEP BY STEP AND INDICATE THE ANSWERS CLEARLY.</u>**

**1.** Use a FULL **truth table** to determine whether the given **argument form** is **valid** or **invalid**. <u>Write a sentence to explain how the truth table supports your answer.</u> (26 points)

$p \vee \sim q$
$r \vee \sim q$
$\sim p$
$\therefore \sim r$

**2.** Prove **directly** if the statements that are true, give **counterexamples** to **disprove** those that are false. (24 points, 12 each)

**(a). "**$\forall\ k, m, n \in \mathbb{Z}^+$, if $k^2 \equiv m^2\ (mod\ n)$, then $k \equiv m\ (mod\ n)$."

**(b).** "$\forall\ m, n \in \mathbb{Z}^+$, if $m > 2$ and $n > 2$ are **prime**, then $m^4 + n^4$ must be **composite**."

**3.** Prove **indirectly**: "$\forall\, n \in \mathbb{Z}, 4 \nmid (n^2 - 2)$." (20 points)

**4.** Prove by **Mathematical Induction** step by step: "$\forall\, n \in \mathbb{Z}^+, 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} \le 2\sqrt{n}$." (30 points)

**5.** Given **sets** $A$ and $B$, define their **symmetric difference** as $A\Delta B = (A - B) \cup (B - A)$. Use the **set properties** to show that $\big(A - (A \cap B)\big) \cup \big(B - (A \cap B)\big) = A\Delta B$, indicate the property used in each step. (15 points)

**6.** Let $\mathbb{Q}$ be the set of all **rational numbers**, for each one of the following two cases, give an **example** of function $f: \mathbb{Q} \to \mathbb{Q}$. Show your examples satisfy the conditions. (20 points, 10 each)

**(a). one-to-one** but **not onto**

**(b). onto** but **not one-to-one**

**7.** Solve the following **counting** and **probability** problems. (15 points, 5 each)

**(a).** How many **permutations** of the letters in word MASSACHUSETTS have consonants in the first two positions and a vowel in the last?

**(b).** A club consists of 6 freshmen, 7 sophomores, and 8 juniors. How many **committees** of 5 people from all classes?

**(c).** What is the **probability** that a five-card poker hand from a standard deck has **two pairs** of **non-face cards**?

**8.** A relation $R$ is defined on the set $\mathbb{R}^+$ of positive real numbers by $xRy$ if the **arithmetic mean** of $x$ and $y$ equals the **geometric mean** of $x$ and $y$, that is, if $\frac{1}{2}(x+y) = \sqrt{xy}$.

**(a).** Prove that $R$ is an **equivalence relation**. (18 points)

**(b).** Describe the distinct **equivalence classes** resulting from $R$. (4 points)

**9.** In each part, either **draw a graph** with the given specifications or **explain why** no such graph exists. (28 points, 7 each)

**(a).** A **graph** with 6 **vertices** that has at least two **circuits**, one is **Euler circuit**, but no **Hamiltonian circuits.**

**(b).** A **connected graph** with **4** vertices that has three **Hamiltonian circuits** but no **Euler circuits**.

**(c).** A **circuit free** graph with 6 vertices and **total degree** 12.

**(d).** A **binary tree** to represent expression $\left(\frac{a-b}{c} + d \cdot e\right) \cdot (f \div (g-h) + i)$.