Question 1:

Consider the penalized formulation for solving a quadratic problem with equality constraints

$$\min_x \frac{1}{2} x^T H x + x^T c + \frac{1}{2\mu} \|Ax - b\|_2^2, \tag{1}$$

where $H \in \mathbb{R}^{n \times n}$ is symmetric, $c \in \mathbb{R}^n$, $\mu > 0$, and $A \in \mathbb{R}^{m \times n}$ with $m < n$ and full row-rank. Let $Z \in \mathbb{R}^{n \times (n-m)}$ be a matrix with orthonormal columns whose range is the null space of A. Show that if $Z^T H Z$ is positive definite, then for all sufficiently small μ (1) has a unique solution and that solution is a strict local minimizer.