6. For $m, n \in \mathbb{N}$, $m \geq 1$, let $S(m, n)$ be the number of solutions to the equation:

$$x_1 + x_2 + \cdots + x_m = n, \text{ where } x_i \in \mathbb{N} \text{ for } i = 1, \ldots, m.$$

Using induction, prove that for all, $m, n \in \mathbb{N}$, $m \geq 1$,

$$S(m, n) = \frac{(n + m - 1)!}{(m - 1)!n!}.$$

[10 Marks]