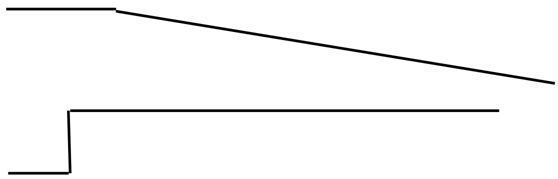
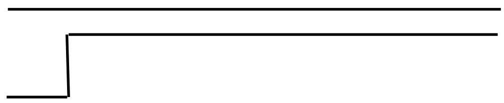
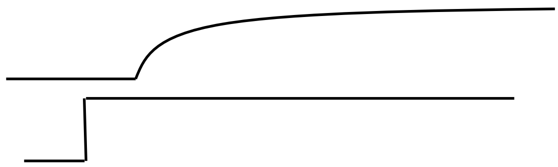
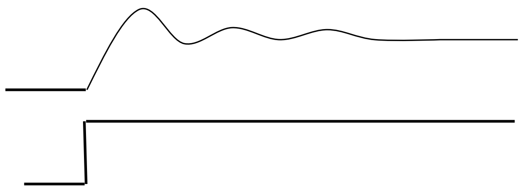
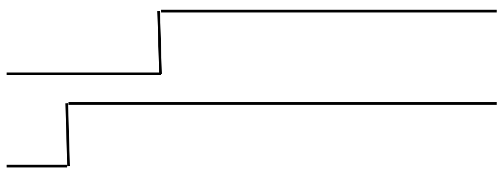
Question 1

Give the (simplest) transfer function that best describes the system that gave the following responses to a step change in the input:

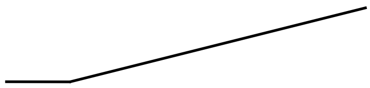
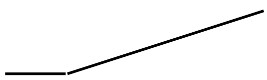
Transfer function that models the system	Output / input
$-\frac{Ke^{-\theta s}}{s}$	
	
	
	
	

(10 marks)

Question 2

In table 2 a transfer function is shown and a sketch of a number of input functions that can be used as the forcing function. Make a sketch of the expected response of the system to the input applied.

Table 2. Reponses of system B to different input functions.

Transfer function that models the system	Input function	Response
$\frac{K}{s}$		
$\frac{K}{s}$		
$\frac{K}{s}$		
$\frac{K}{s}$		
$\frac{K}{s}$		

(7 marks)

Question 3

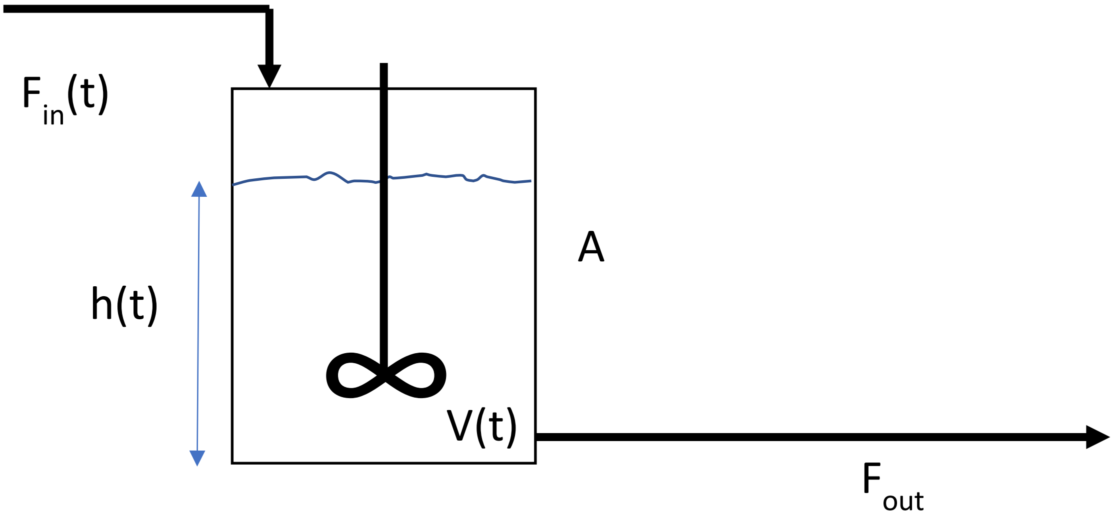


Figure 3. A buffer tank.

See Figure 3. A buffer tank with cross section A has a variable amount of liquid $V(t)$ with density ρ in it. The inflow of liquid $F_{in}(t)$ into the tank varies, but the outflow F_{out} is constant.

a) Set up a balance for the amount of liquid in the tank.

b) Determine the transfer function $G(s)$ describing the dependence of the liquid level in the tank on the inflow, i.e. $G(s) = H(s)/\Phi(s)$.

c) Derive an equation describing how the level in the tank changes in time after a step $\Delta F u(t)$ in the inflow.

(6 marks)

Question 4

A process fluid is cooled down in a continuously stirred vessel with a cooling jacket as shown in Figure 4. It can be assumed that the volume of liquid V in the tank is constant. The process stream has a heat capacity C_p, and density ρ. The specific heat capacity of a substance C_p is the amount of heat ΔH that must be added to one unit of mass of the substance in order to cause an increase of one unit in temperature.

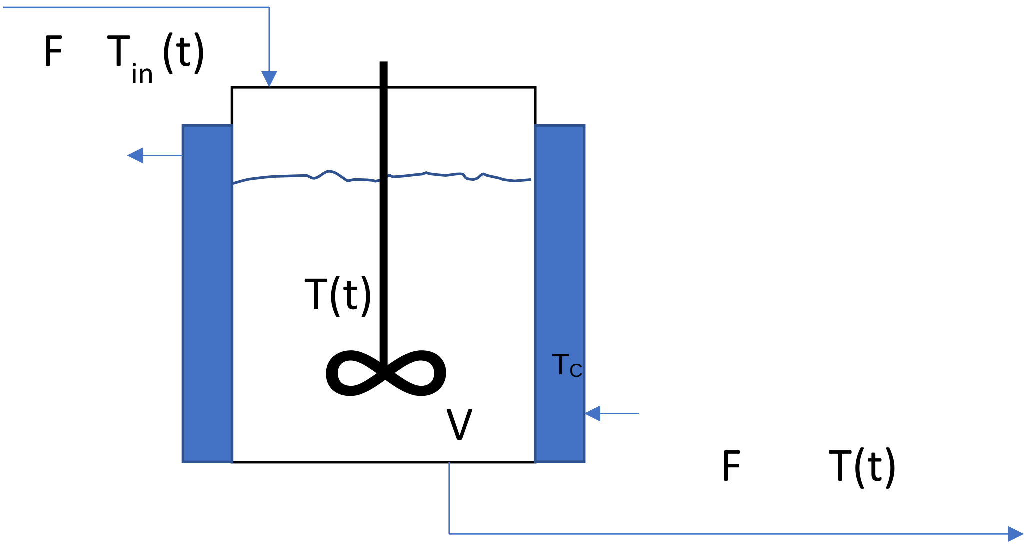


Figure 4. Cooling of a process stream with a jacketed stirred vessel.

The amount of heat transferred during cooling is described by $UA(T(t)\text{-}T_c)$, in which U is the heat transfer coefficient, A the area of heat transfer, $T(t)$ the temperature in the vessel, and T_c the temperature of the coolant in the jacket. T_c can be assumed to be constant.

a) Set up an energy balance for the vessel.

b) Define deviation variables and determine the transfer function describing the dependence of the output temperature T on the temperature of the inflow T_{in}.

(10 marks)

Question 5

A product X is made by the degradation of a compound A in the irreversible reaction

A $\rightarrow$ X

The reaction rate is only dependent on the concentration of reactant A and is first order i.e.

$r_A = -\frac{dC_A}{dt} = k_1 C_A$

The reaction is exothermic (i.e. heat is produced during the reaction). The amount of heat produced is proportional to the amount of X produced and hence to the amount of A consumed. For every kg of A consumed ΔH_R heat is produced.

It is proposed to make compound X in a continuous reactor with constant volume *V* as shown in Figure 5. There is no compound X in the inflow. It can be assumed that the reactor is well insulated, and no heat is lost to the environment. The reactor is well-mixed, and it can be assumed that the temperature and concentrations are the same throughout the reactor (including the outlet)

It can be assumed that the heat capacity C_p for the inflows and outflows are the same. It can also be assumed that the density ρ of the inflow and outflow are the same.

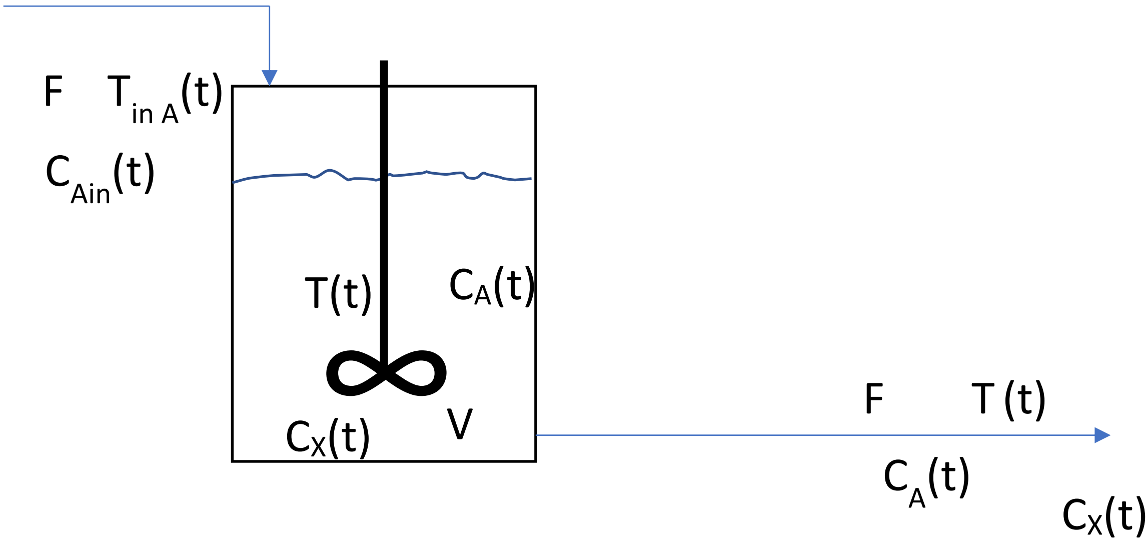


Figure 5. Proposed process for the production of X.

a) Write a mass balance for compound A.

b) Write a mass balance for compound X.

c) Write an energy balance for the temperature $T(t)$ in the tank.

Do not attempt to derive transfer functions.

(12 marks)

Question 6

Linearise the following ODEs:

$$a_0 \frac{dy(t)}{dt} = b_1 x_1(t) x_2(t) - b_2 x_3(t) + b_3$$

$$a_1 \frac{dy(t)}{dt} = c_1 [x_1(t)]^3 - c_2 x_2(t) - c_3 [x_3(t)]^2$$

(7 marks)

Question 7

Derive the transfer functions $Y(s)/X_1(s)$ and $Y(s)/X_2(s)$ for the following ODE:

$$a_0 \frac{dy(t)}{dt} + a_1 y(t) = b_1 [x_1(t)]^2 + b_2 x_2(t) + b_3$$

(9 marks)

Process Modelling and Control Data Sheet

The Laplace transforms of an equation $f(t)$ is defined as:

$$\mathcal{L}[f(t)] = F(s) = \int_0^\infty f(t)\exp(-st)dt$$

Table 1. Laplace transform of common functions:

Function $f(t)$	Laplace transform $F(s) = \mathcal{L}[f(t)]$
Dirac function $\delta(t)$	1
Unit step $u(t)$	$\frac{1}{s}$
a	$\frac{a}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\exp(-at)$	$\frac{1}{s+a}$
$t\exp(-at)$	$\frac{1}{(s+a)^2}$
$t^n \exp(-at)$	$\frac{n!}{(s+a)^{n+1}}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$\exp(-at)\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$\exp(-at)\cos(\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

Table 2. First order system

Transfer function	$Y(s) = \left[\frac{K}{\tau s+1}\right]X(s)$
Root	$r = -1/\tau$

Table 3. Second order system

Transfer function	$Y(s) = \left[\frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}\right]X(s)$
Roots	$r_1 = \frac{-\zeta - \sqrt{\zeta^2 - 1}}{\tau}$; $r_2 = \frac{-\zeta + \sqrt{\zeta^2 - 1}}{\tau}$
Period of oscillation	$T = 2\pi/\omega$
Damped oscillation:	
Rise time	$T/4 = \pi/2\omega$
Decay ratio	$= \exp(-(\zeta/\tau)T)$
Overshoot	$= \exp(-(\zeta/\tau)T/2)$
Settling time	$= 5\tau/\zeta$