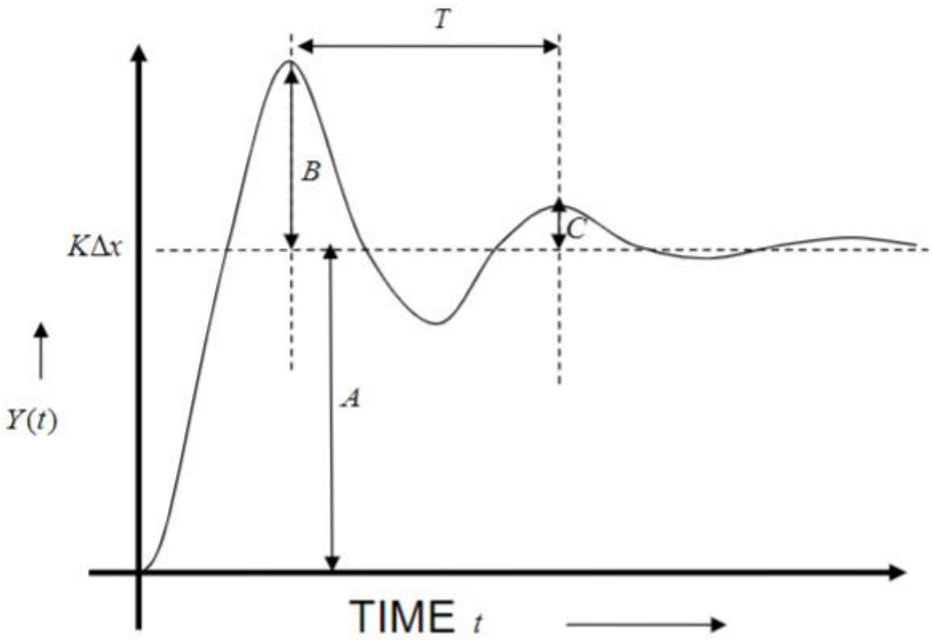
2.7 Characterising an Underdamped Response

Underdamped responses are commonly found in control systems and it is, therefore, useful to study them in more detail:



Various parameters are used to characterise an underdamped response. One is the "period of the oscillation" T. This is inversely related to the frequency ω through the expression

$$T = \frac{2\pi}{\omega} \qquad (3.30)$$

The frequency was previously shown to depend on the damping ratio ζ, so that the above may also be written as

$$T = \frac{2\pi\tau}{\sqrt{1-\zeta^2}} \qquad (3.31)$$

The “rise time” is the time it takes for the output to first cross over its final steady state value

$$\text{Rise Time} = \frac{T}{4}$$

Through equation (3.31), the rise time may also be expressed in terms of the damping ratio as follows:

$$\text{Rise Time} = \frac{\pi\tau}{2\sqrt{1-\zeta^2}} \quad \text{..................................(3.32)}$$

The degree of oscillation is characterised by the decay ratio C/B, which is the ratio of successive “peak overshoots” above the final steady-state value (see previous diagram)

$$\text{Decay Ratio} = \frac{C}{B}$$

$$\text{Decay Ratio} = \exp\left(-(\zeta/\tau)T\right)$$

$$\text{Decay Ratio} = \exp\left(-2\pi\zeta/\sqrt{1-\zeta^2}\right) \quad \text{...................(3.33)}$$

The “overshoot” is the fraction the first peak exceeds the final steady state change , i.e. B/A (see previous diagram)

$$\text{Overshoot} = \frac{B}{A} = \exp\left(-(\zeta/\tau)T/2\right)$$

$$\text{Overshoot} = \exp\left(-\pi\zeta/\sqrt{1-\zeta^2}\right) \quad \text{.......................(3.34)}$$

The settling time is the time it takes for the response to come within a certain band around the final steady-state value. The output will have settled within 1% of its final value after settling time of approximately

$$\text{Settling Time} = 5\frac{\tau}{\zeta} \quad \text{......................................(3.35)}$$