Problems

1. Solve the differential equation by an appropriate substitution $x\frac{dy}{dx} + 6y - x^6y^2 = 0$.

2. Show that the following differential equation is homogeneous and solve it by an appropriate substitution $x\frac{dy}{dx} = y + \sqrt{x^2 - y^2}$, for $x > 0$.

3. Solve the differential equation $\frac{dy}{dx} = \frac{x + y - 1}{x + y}$.

4. Solve the differential equation

$$(2y \sin x \cos x - y + 2y^2 e^{xy^2})dx + (-x + \sin^2 x + 4xye^{xy^2})dy = 0 \ .$$

5. Solve the differential equation $(x^2 - 1)\frac{dy}{dx} + 2y = (x + 1)^2$.

6. This question uses MATLAB. The differential equation

$$\frac{dy}{dx} = \frac{-x + \sqrt{x^2 + y^2}}{y} \qquad (1)$$

describes the shape of a plane curve C that will reflect all incoming light beams to the same point and could be a model for a mirror of a reflecting telescope, a satellite antenna, or a solar collector. Solve this differential equation as follows.

(a) Verify that the differential equation is homogeneous. Show that the substitution $y = ux$ leads to

$$\frac{udu}{\sqrt{1 + u^2}(1 - \sqrt{1 + u^2})} = \frac{dx}{x} \ .$$

Use the substitution $w = 1 - \sqrt{1 + u^2}$ and solve the resulting DE to find $w = w(x)$. Then utilize it to find the solution of the differential equation (1). Show that the curve C is a parabola with focus at the origin and is symmetric with respect to the x-axis.

(b) Verify your answer for (a) by using the following commands to solve the differential equation (1). Provide the solutions with real values given by MATLAB.

```
clear all; % clear the workspace
syms y(x) % create symbolic variables x and y(x)
eqn = diff(y,x) == -(x+ sqrt(x^2+y^2))/y; % define the differential equation
GenSol(x) = dsolve(eqn) % solve for the general solution
```