

Answer all four questions

1. A company produces a closed rectangular crate with volume $0.5 m^3$. Suppose that the length, breadth and height of the box are given by x , y and z respectively, and that the cost of materials is $\pounds 5$ per square meter for the bottom, $\pounds 1$ per square meter for the sides, and $\pounds 3$ per square meter for the top. Show that the cost of the crate in pounds can be expressed as

$$C(x, y) = 8xy + \frac{1}{y} + \frac{1}{x}.$$

Hence find the cost of the cheapest crate.

(Once you find a candidate minimum you are not required to show that it is in fact a minimum.)

[10 marks]

2. Find all the stationary points, and determine their nature, for the function

$$f(x, y) = y^3 - 6xy + 6x^2.$$

[10 marks]

3. (a) If δx , δy are errors in the quantities x and y , and the function $f(x, y)$ is given by

$$f(x, y) = xy^3$$

then use Taylor's series to derive a linear approximation for the associated error δf .
What are the relative and percentage errors in f ? [5 marks]

- (b) The power, P , in an AC circuit is given by $P = I^2 R$, where I is the current and R is the resistance. Find the percentage error in P if the percentage error in I is 2% and the percentage error in R is 1%. [5 marks]

4. (a) Evaluate the double integral

$$I = \int_1^2 \int_1^2 y/x \, dydx.$$

[5 marks]

- (b) Evaluate the double integral

$$I = \int_0^1 \int_0^{y^2} (x + e^{y^3}) \, dx dy.$$

[5 marks]

END