ANSWER ALL FOUR QUESTIONS

1. Solve the following initial value problem for an inhomogeneous 2nd order ODE:

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 3\frac{\mathrm{d}y}{\mathrm{d}t} - 4y = 5e^t\ , \qquad y(0) = 0\ , \quad \frac{\mathrm{d}y}{\mathrm{d}t}(0) = 1\,.$$

[10 marks]

2. (New type of ODE) Find the general solution, or at least some solutions, to the ODE

$$t\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 2\frac{\mathrm{d}y}{\mathrm{d}t} = -y(t)t.$$

Please explain your reasoning in detail. Hint: consider the function $z(t) = y(t)t$.

[10 marks]

3. Find the partial derivatives $\dfrac{\partial f}{\partial x}$, $\dfrac{\partial f}{\partial y}$, $\dfrac{\partial^2 f}{\partial x^2}$, $\dfrac{\partial^2 f}{\partial y^2}$, and $\dfrac{\partial^2 f}{\partial x \, \partial y}$ for

$$f = \ln(x + y) + xy^3 \, .$$

[10 marks]

4. The position $x(t)$ of a damped oscillating particle of mass m at time t on a spring acted on by an external force $F(t)$ can be represented by the following differential equation:

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + c\frac{\mathrm{d}x}{\mathrm{d}t} + kx = F(t)$$

where c is the damping coefficient and k is the spring constant. Find the general solution for $m = 2$ and $c = k = 1$ and $F(t) = \sin(t)$.

What is the amplitude and frequency of the motion $x(t)$ as $t \to \infty$?

[10 marks]

END