Assignment 4

Note: This assignment covers some of the topics of Unit 5 and Unit 6. You have to show all your work in the answers in order to obtain full marks. For your convenience, each exercise mentions the section of the *Study Guide* that needs to be studied before solving the problem. Use exact answers for all questions. **No** numerical approximations (a number that has a decimal point in it, for example, 2.29) are allowed anywhere.

Instructions for submitting your assignments are found on the course home page under the *Assessments* section.

Total points: 100
Weight: 10%
Due: After Unit 6

1. **(8 points)** 🕮 Sections 5.1–5.2

   a. Show that the sequence $\dfrac{9^n}{n!}$ converges.

   b. Show that the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges.

2. **(16 points)** 🕮 Sections 5.4–5.5

   Determine the appropriate Taylor series expansion for the following functions.

   a. $\log(z+21),\ z_0 = 0$

   b. $z^2e^z - 2ze^z + e^z,\ z_0 = 1$

   c. $z\sin z^2,\ z_0 = 0$

   d. $\cosh z,\ z_0 = 0$

3. **(16 points)** 🕮 Sections 5.4–5.5

   Use the ratio test to determine the radius of convergence (denote by R) in each of the cases in the previous question.

4. **(5 points)** 🕮 Section 5.4–5.5

   Determine the first three terms in the Taylor series expansion of $\sqrt{z^2-1}$ about $z_0 = 0$ by using Newton's binomial formula (Newton's generalized binomial theorem). Choose the branch of square root so that $\sqrt{-1} = i$. What is the radius of convergence?

5. **(24 points)** 🕮 Section 5.6

Determine the following Laurent series.

a. $\dfrac{1}{z-i} + \dfrac{1}{z-1}$ on

i. $\{z \in \mathbb{C} | 0 < |z-1| < \sqrt{2}\}$

ii. $\{z \in \mathbb{C} | |z-1| > \sqrt{2}\}$

b. $\dfrac{z+2}{z-1}$ on

i. $\{z \in \mathbb{C} | 0 < |z| < 1\}$

ii. $\{z \in \mathbb{C} | |z-1| > 1\}$

c. $\dfrac{z}{(z-1)(z+4)}$ on

i. $\{z \in \mathbb{C} | 0 < |z-1| < 5\}$

ii. $\{z \in \mathbb{C} | |z-1| > 5\}$.

6. **(16 points)** 🕮 Sections 6.1–6.3

For each of the singularities of the following functions:

i. find the residues

ii. find the principal parts

iii. classify the singularities as one of the types: removable, pole or essential.

a. $\dfrac{\sin z^2}{z^4}$

b. $\dfrac{\cosh z}{z^2}$

c. $\dfrac{z+2}{z^2-3z}$

d. $ze^{\frac{1}{z}}$

7. **(15 points)** 🕮 Sections 6.4–6.6

Use the residue theorem to calculate the following integrals.

a. $I_1 = \displaystyle\int_0^\infty \frac{dx}{x^2+9}$

b. $I_2 = \displaystyle\int_0^\infty \frac{dx}{(x^2+1)^2}$

c. $I_3 = \displaystyle\int_0^\infty \frac{\cos 2x\, dx}{x^2+4}$

d. $I_4 = \displaystyle\int_{-\infty}^\infty \frac{2i\sqrt[3]{x} + e^{2ix}\, dx}{x^2+4}$