Assignment 3

Note: This assignment covers some of the topics of Unit 4. You have to show all your work in the answers in order to obtain full marks. For your convenience, each exercise mentions the section of the *Study Guide* that needs to be studied before solving the problem. Use exact answers for all questions. **No** numerical approximations (a number that has a decimal point in it, for example, 2.29) are allowed anywhere.

Instructions for submitting your assignments are found on the course home page under the *Assessments* section.

Total points: 100
Weight: 10%
Due: After Unit 4

1. **(10 points)** 🕮 Sections 4.1–4.2

   Evaluate the following integrals.

   a. $\int_C (3x - 2i)^2\, dx + dy$, where C is the line segment $y = 2x, 0 \leq x \leq 2$

   b. $\int_0^{115\pi} e^{it}\, dt$

2. **(24 points)** 🕮 Sections 4.2–4.4

   a. Compute $\int_C f$ for the given functions and contours.

      i. $f(z) = x^2 + iy^2, C : y = x^3, -1 \leq x \leq 1$

      ii. $f(z) = 3x^2 - 2iy, C : y = x^3, 0 \leq x \leq 1$

      iii. $f(z) = \bar{z}, C$: straight line segments from 0 to 1 then to $1 + i$

      iv. $f(z) = \bar{z}, C$: straight line segments from 0 to $1 + i$ directly

   b. Explain why the results in (iii) and (iv) are different.

3. **(10 points)** 🕮 Section 4.2

   Page 169 of the textbook says that if $|f(z)| \leq M$ for all z on a (piecewise smooth) curve C with length L, then $\left|\int_C f\right| \leq M \cdot L$. Now let C be the upper half circle $|z| = R > 2, \mathrm{Im} z > 0$, show that

$$\lim_{R\to\infty} \int_C \frac{2z^2 - 5}{(z^2+1)(z^2+4)}\, dz = 0.$$

4. **(10 points)** 📖 Section 4.3

   Show that $\int_C f = 0$, where $C : |z| = \sqrt{2}$ and

   a. $f(z) = \dfrac{(z^2+2)}{(z^2+3)}$.

   b. $f(z) = \dfrac{(z+\sin z-2)}{\cos z}$.

5. **(10 points)** 📖 Section 4.3

   Show that $\int_{C_1} f = \int_{C_2} f$, where $C_1 : |z| = 1$ and $C_2 : |z| = 2$ and

   a. $f(z) = \dfrac{(2z^2+3z+4)}{\sin z}$.

   b. $f(z) = \dfrac{(z^2+i)}{(e^z+1)}$.

6. **(10 points)** 📖 Section 4.5

   Evaluate the following integrals:

   a. $\displaystyle\oint_C \frac{(\tan z+1)}{(z^2-z)}\,dz$, where $C : |z| = 1/2$

   b. $\displaystyle\oint_C \frac{(\cos z+e^z)}{(z-1)^4}\,dz$, where $C : |z-1| = 1/2$

7. **(8 points)** 📖 Section 4.6

   Evaluate the following integral by using Causs' Mean Value Theorem.

   $$\int_0^{2\pi} \sec(e^{i\theta})\,d\theta.$$

8. **(8 points)** 📖 Section 4.6

   Let $f(z) = z^7$ and R be the region $|z+1+i| \le \sqrt{2}$. Find the value(s) of z in R where $|f(z)|$ obtains its maximum and minimum values, evaluate the maximum and minimum values of $|f(z)|$ at the points you find.

9. **(10 points)** 📖 Section 4.6

   Give an example of a function $f(z)$ that is analytic in a region, bounded throughout the complex plane but is not a constant function. Does this contradict the Liouville's theorem? Explain.