1. **(8 points)** Section 3.1

   a. If $e^{iz} \in \mathbb{R}$, find all possible values of z.

   b. If e^{z} is purely imaginary (can be written in the form of iy, y is non-zero real number), find all possible values of z.

2. **(8 points)** Section 3.1

   Write $|e^{5z+3i+2}|$ and $|e^{iz^2}|$ in terms of x, y, where $z = x + iy, x, y \in \mathbb{R}$ and then show that

   $$|e^{5z+3i+2} + e^{iz^2}| \leq e^{5x+2} + e^{-2xy}.$$

3. **(20 points)** Sections 3.2–3.4

   a. Show that $|\sin z|^2 = \sin^2 x + \sinh^2 y$.

   b. Use part (*a*) to show that the complex sine function is not bounded: for any $M > 0$, there exist a z such that $|\sin z| > M$.

   c. Solve the equation $\cos z = 3$.

   d. For one root of (choose any root you wish) the above equation, verify that

   $$\sin^2 z + \cos^2 z = 1.$$

4. **(12 points)** Sections 3.1–3.2

   Solve each of the following for $z \in \mathbb{C}$. Do ***not*** use any formula for inverse functions in Section 3.7.

   a. $\sin z = 0$.

   b. $\tan \pi z = 1$.

   c. $1 + e^{2z} = 0$.

5. **(10 points)** Section 3.4

   Solve the following equations.

   a. $\text{Log}(z^2 - 1) = \dfrac{i\pi}{2}$

   b. $e^{2z} + e^z + 1 = 0.$

6. **(20 points)** Sections 3.1–3.5, 3.7

   Differentiate the following functions, state the regions where the functions are analytic.

   a. $\cos(e^z)$

   b. $\dfrac{1}{e^z + 1}.$

   c. $\text{Log}\,(z^2 + 1)$ (***Hint:*** To find where it is analytic, you may let $z = x + iy$.)

   d. $\tanh^{-1}(z)$, use $z = 0$ and the negative real axis as branch cut for the logarithmic function involved. For calculating the derivative, you need to show intermediate steps, since the answer is already given in the textbook.

7. **(10 points)** Sections 3.6–3.7

   Evaluate the following and write your answer in such a way that the real and imaginary part can be identified clearly (polar form or exponential form will be fine).

   a. i^{i^i}, note that this is ***not*** $(i^i)^i$. You can use this result: $i^i = e^{-\frac{\pi}{2}-2k\pi}$, $k \in \mathbb{Z}$ (the negative sign in front of k is just a personal preference, you can use plus sign if you wish, since k can be negative).

   b. $\tan^{-1}(1)$, using the formula $\tan^{-1} z = \dfrac{i}{2} \log\left(\dfrac{i+z}{i-z}\right).$

8. **(12 points)** Section 3.8

   Find a conformal map that maps the horizontal strip $\text{Im}z \in (0, \pi)$ onto the unit disk $|z| < 1$ and maps $\dfrac{\pi}{2}i$ to 0.