Assignment 4

Note: This assignment consists of 10 problems of equal weight.

Due: After Unit 15

1. Given that $x + 2$ is a solution to the homogeneous part of the equation

   $$xy'' - (x + 2)y' + y = 2x^3e^x,\ 0 < x,$$

   find the general solution to the equation as a whole.

2. Reduce the order of the following differential equation and solve

   $$2x^2z''' + xz'' - 3z' = \frac{2}{x^3},\ x > 0.$$

3. Find at least the first four nonzero terms in a power series expansion of the solution to the IVP

   $$y'' - y' \sin x - y = \cos x,\ y(\pi) = 0,\ y'(\pi) = 1.$$

4. Find the first four nonzero terms in a power series expansion about $x = 0$ for each of two linearly independent solutions and the particular solution to

   $$y'' - (x + 1)y = 1.$$

   Then compile the general solution.

5. For the equation given below, classify all singular points as regular or irregular. For those that are regular, compile indicial equations, find roots, and write theoretic forms of the first and the second series solutions to the equations. [It is not necessary to solve the equations.]

   $$(x^3 - 5x^2 + 6x)y'' + 3\frac{y'}{x - 2} - 2\frac{x + 3}{x - 3}y = 0.$$

6. Find all terms of both linearly independent solutions to the equation

   $$2x(x - 1)y'' - (3x + 1)y' + 2y = 0$$

   by the series method at point $x = 0$.

7. Given the first linearly independent solution $y_1(x) = x$ and the point of expansion $x_0 = 0$, find the second linearly independent solution $y_2(x)$ to the equation

   $$x(x + 1)y'' + xy' - y = 0$$

   by the method of Frobenius.

8. Given the autonomous system

$$\frac{dx}{dt} = x, \ \frac{dy}{dt} = x^2 + y,$$

   a. find the critical points.

   b. solve the phase plane equation and classify the critical points, analyzing trajectories of the system.

   c. find the general solution to the system and determine its stability, analyzing integral curves of the system.

9. For each of the linear systems given below, find all the critical points in the plane, and analyze the stability of each point.

   a. $\frac{dx}{dt} = x - 1, \ \frac{dy}{dt} = 2x + y + 1.$

   b. $\frac{dx}{dt} = x + 2y, \ \frac{dy}{dt} = 2x + 5y - 2.$

10. Determine the type and stability of the critical points of the following autonomous system:

$$\frac{dx}{dt} = x^2 - 1, \ \frac{dy}{dt} = 2 - 2x - y^2.$$