

Assignment 3

Note: This assignment consists of 5 problems of equal weight.

Due: After Unit 11

1. Find Laplace transforms for the following function

$$f(t) = \begin{cases} 1, & \text{if } 0 < t < 1, \\ -2, & \text{if } 1 < t < 2, \\ t^2, & \text{if } 2 < t. \end{cases}$$

2. Find the partial fraction expansion for:

$$P = \frac{1 - 3s^3}{(s^3 - 4s)(s^2 - 1)}$$

and, on this basis, find the inverse Laplace transform, $\mathcal{L}^{-1}\{P\}$.

3. Find

$$\mathcal{L}^{-1}\left\{\frac{s^2}{(s^2 + 1)^2}\right\}$$

using the convolution theorem.

4. Solve the following initial value problem

$$\begin{cases} x'(t) + y(t) = 2 \\ y'(t) - x(t) = \delta(t - \pi) \\ x(0) = 0, y(0) = 1. \end{cases}$$

5. Given one initial condition $y(0) = 0$, find a value of parameter a such that the solution of the IVP

$$y''(t) + 9y(t) = 8\delta\left(t - \frac{\pi}{2}\right), \quad y(0) = 0, \quad y'(0) = a,$$

satisfies the condition $y\left(\frac{\pi}{2}\right) = 1$.