1. With justification, determine the points in $[0, \pi]$ at which $f(x) := e^{2\sin x - x}$ attains its maximum and miniumn values.

2. (a) Find the linearisation of $f(x) = \sqrt[3]{x}$ at $a = 8$.

   (b) Use this linearisation to approximate $\sqrt[3]{7}$ and $\sqrt[3]{9}$.

   (c) Using a calculator, show that the difference between the approximate and true values of the quantities in 2(b) is strictly less than 0.004.

3. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for each of the following functions $f : \mathbb{R}^2 \to \mathbb{R}$.

   (a) $f(x, y) = x^2 e^y$

   (b) $f(x, y) = xy^2 - x \sin y$.

4. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x, y) = 8x^3 - 9x^2 + 6xy + 3y^2 + 11.$$

   (a) Find ∇f.

   (b) Solve $\nabla f = (0, 0)$ (**hint:** there are two critical points).

   (c) Determine the Hessian matrix $H_f(x, y)$ at each point $(x, y) \in \mathbb{R}^2$.

   (d) Use Theorem 5.16 in the notes to classify the critical points of f.

5. Find and classify the critical points of the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x, y) = x^3 + 3xy^2 - 12x + 6y.$$

6. In this question we use the method of Least Squares to find the line of best fit $f(x) = mx + c$ through the data

$$(5, 7), \quad (7, 10), \quad (10, 18) \quad \text{and} \quad (11, 20).$$

   (a) Establish the discrepancies d_1 to d_4 and write the sum of their squares as a function g of m and c.

   (b) Find the line of best fit by solving $\nabla g = 0$. You do not need to verify that your solution is a local minimum of g.

   (c) Using your line of best fit, estimate the location of the data point having x-coordinate 15.