1. Let the straight line L in $\mathbb{R}^n$ pass through the origin $\mathbf{0}$, parallel to a non-zero vector $\mathbf{v}$. Let $\mathbf{x}$ be another vector in $\mathbb{R}^n$.

   (a) Using Example 2.31, show that $\|\mathrm{proj}_L(\mathbf{x})\| = \dfrac{|\mathbf{x} \cdot \mathbf{v}|}{\|\mathbf{v}\|}$.

   (b) Calculate $\mathrm{proj}_L(\mathbf{x})$ and $\|\mathrm{proj}_L(\mathbf{x})\|$ when $n = 3$, $\mathbf{v} = (-2, 1, 4)$ and $\mathbf{x} = (7, 0, 11)$.

2. Solve the following linear systems:

   (a)
$$\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 8 \\ 2x_1 + 3x_2 - 7x_4 &= 8 \\ x_1 + 7x_2 + 4x_3 - 14x_4 &= 3, \end{aligned}$$

   (b)
$$\begin{aligned} x_1 - 2x_2 + 3x_3 - 4x_4 &= 5 \\ 2x_1 - 4x_2 + 8x_3 - 2x_4 &= 20 \\ -3x_1 - 6x_2 - 7x_3 + 18x_4 &= -5. \end{aligned}$$

   (c)
$$\begin{aligned} x_1 - x_2 + 3x_3 - 2x_4 &= 3 \\ 2x_1 - 9x_2 - 7x_3 + x_4 &= 13 \\ -2x_1 + 2x_2 - 6x_3 + 4x_4 &= -5. \end{aligned}$$

6. Let $v_1 = (\frac{4}{13}, \frac{12}{13}, -\frac{3}{13})$, $v_2 = (\frac{12}{13}, -\frac{3}{13}, \frac{4}{13})$ and $v_3 = (\frac{3}{13}, -\frac{4}{13}, -\frac{12}{13})$.

(a) Verify that v_1, v_2, v_3 is an orthonormal basis of $\mathbb{R}^3$.

(b) In both cases below, determine the three coordinates of the given vector x with respect to v_1, v_2, v_3.

i. $x = (1, -2, 1)$

ii. $x = \sqrt{5}v_2 - \pi^2 v_3 + 10^{102} v_1$.