4. Two unit vectors x and y in $\mathbb{R}^n$ satisfy $x \cdot y = \frac{\sqrt{3}}{2}$. Evaluate

(a) the angle between x and y, in radians

(b) $y \cdot (7x + 3y)$

(c) the length of $2x - 9y$.
Hint: You do not need to use the cosine rule to solve this part.

5. Let $x = (0, 1)$ and $y = (-1, a)$ be two vectors in $\mathbb{R}^2$, where a is a real number.

(a) Compute the quantity $\frac{x \cdot y}{\|x\| \|y\|}$, in terms of a.

(b) Determine all values of a for which the angle between x and y is $\frac{\pi}{3}$ (radians).

6. Let x and y be non-zero vectors in $\mathbb{R}^n$.

(a) Suppose that $\|x + y\| = \|x - y\|$. Show that x and y must be perpendicular.

(b) Suppose that $x + y$ and $x - y$ are non-zero and perpendicular. Show that x and y must have the same length.

Note: you are not allowed to choose specific non-zero vectors x and y and solve for them; your solutions have to work for **all** such vectors.