Assignment 1

Note: This assignment consists of 10 problems of equal weight.

Due: After Unit 6

1. Solve the following initial value problem,

$$y'(xy + y - 2x - 2) = \ln(x+1),\ y(0) = 0,\quad x > -1.$$

2. Find a special integrating factor and solve

$$xyy' + x + y^2 = 0.$$

3. Find an integrating factor and solve

$$(2x^2y + x)dy + (xy^2 + y)dx = 0.$$

4. Solve the following initial value problem,

$$x\,dy + (y - y^2 \ln x)\,dx = 0,\ y(1) = \frac{1}{4}.$$

5. Solve

$$2x\frac{dy}{dx} - y = y\left[1 - \ln^2\left(\frac{y}{x}\right)\right],\quad x > 0.$$

6. Solve

$$\frac{dy}{dx} - \cos^2(x - y) = 0.$$

7. Solve

$$(y')^2 + (x + 2y)\cos(x + y) = (x + 2y + \cos(x + y))y'.$$

8. A tank is filled with $V = 200$ L of a brine containing $\alpha = .4$ kg of salt *A* per litre. At moment 0, input and output valves are opened, and a brine containing another salt *B*, with concentration $\beta = .2$ kg per litre runs into the tank at a rate $r_{\mathrm{i}} = 5$ L/sec. The mix runs out of the tank with rate $r_{\mathrm{o}} = 4$ L/sec. The salts do not interact with each other. Determine the ratio k of quantity of salt *B* to the quantity of salt *A* when the tank contains $V_1 = 400$ L of the mixture.

9. (Heating)

The temperature $M(t)$ outside a building decreases at a constant rate of $1°$C per hour. The inside of the building is heated, and there is no other source of cooling. The heater was switched on at time $t = 0$, when the temperature inside, $T(t)$, was $17°$C, and the temperature outside was $0°$C. Assume that the heater generates a constant amount $h = 50{,}000$ Btu/hr of heat when it is working, the heat capacity of the building is $\gamma = 1/5$ degrees per thousand Btu, and the time constant for heat transfer between the outside and the inside of the building is $\tau = 2$ hr. On the basis of Newton's law of cooling,

$$\frac{dT(t)}{dt} = K(M(t) - T(t)) + \gamma h,$$

find the upper value of the temperature in the building in the time interval $0 \leq t < 4$ hr.

10. (Landing)

    A container with mass M kg is dropped by a helicopter from height H km at time $t = 0$, with zero velocity. From the outset, its fall is controlled by gravity and the force of air resistance, $f(v) = -kv$, where v is the current velocity of the container.

    In τ seconds after the drop, a parachute opens, resulting in an increase of air resistance up to $F(v) = -Kv$. Determine the time T at which the container touches the ground, and its velocity at this moment, if

    $$M = 200 \text{ kg}, \ H = 2000 \text{ m}, \ \tau = 20 \text{ s}, \ k = 10 \text{ kg/s}, \ \text{ and } \ K = 400 \text{ kg/s}.$$