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DYNAMIC PRICE MODELS FOR NEW-PRODUCT PLANNING*

BRUCE ROBINSON† AND CHET LAKHANI§¶

The major points established in this paper are: classic marginal pricing is far from optimum for a rapidly evolving business; more appropriate dynamic models can be formulated if one has some feeling for the dominant evolutionary forces in the business environment; and, planning based on the dynamic models can lead to a significant improvement in the long run profit performance. Two developments in the management science literature, the experience curve phenomenon and market-penetration models, are used to illustrate the nature of the dynamic feedback between market and production activity which causes a new growth business to evolve. A specific illustrative example is offered which demonstrates the fact that dynamic price models can be used to test the long run consequences of specific pricing rules or to determine the optimum long run pricing scenario within the context of any constraints which a manager might wish to impose. As opposed to the conventional static theory which emphasizes the instantaneous profit flow, the dynamic models use an appropriately discounted accumulated profit as the major parameter for making value judgments. The specific example considered emphasizes the importance of these ideas for a growth market and suggests that dynamic models can lead to as much as an order of magnitude more profit in the long run than the conventional static theory. More modest, but significant, improvements in long run performance can be obtained in a moderate growth business.

1. Introduction

One of the major shortcomings of conventional price theory is its focus on the short term. It assumes static market and production environments and uses the instantaneous profit flow as a key parameter for making value judgments. One of the reasons for its neglect in the real world is the fact that managers are acutely aware of the fact that their market and production environments are evolving with time. They resort to pricing rules which they intuitively feel will produce better performance in the long run.

There have, however, been developments in the management science literature which open up the possibility of making a formal analysis of long run pricing strategies which is pertinent to certain business situations and illustrative of a general approach to pricing for the long run. The resulting dynamic approach permits one to seek an optimum price strategy within the context of any production or market constraints or to determine the long run consequences of any "practical" pricing rule-of-thumb. Both the conventional static and the new dynamic models are applied to a specific case in §6.

2. Conventional Price Theory

The basic inputs of conventional price theory are a demand schedule which describes the market by relating the sales volume, V, to the unit price, P, and a cost

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† RCA Laboratories, Princeton, New Jersey.

§ RCA Consumer Electronics, Indianapolis, Indiana.

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schedule which relates the unit cost, C, to the rate of production, V.

$$V = V(P) \tag{1}$$

$$C = C(V) \tag{2}$$

Since we are concerned with the long run, we shall assume that the sales volume equals the rate of production. If the inventory is held constant, this is also true in the short run. The major parameter for making value judgments is the instantaneous profit flow, $(P - C)V$. The instantaneous profit flow is maximized by the marginal pricing rule,

$$\text{Marginal Revenue} = \text{Marginal Cost}$$

(3) or

$$\partial[PV]/\partial V = \partial[CV]/\partial V.$$

This basic model has been modified over the years to include the complications of promotion costs, multiple products and competitive gaming ([5] and [3]) but, it has remained an essentially static analysis which uses instantaneous profit flow as a major parameter for value judgments.

3. Dynamic Price Models

Since most corporations are in business for the long run, a major parameter in any price model should be the integrated profit obtained throughout some appropriate planning period. Estimation of this parameter requires some knowledge of the possible evolution of the cost and demand schedules. The major point of this paper is that a manager who has some insight into how his costs and markets are going to evolve (not necessarily the ones used here for illustrative purposes) can incorporate his ideas into a dynamic pricing model which, for a rapidly evolving business, can greatly enhance his long run performance. Our discussion is meant to illustrate a general approach to the problem of pricing in a rapidly evolving environment.

In the following two sections, we shall review two illustrative models from the management science literature which indicate the kind of cost and demand evolution which is appropriate for some businesses. Both examples suggest that the evolution of a new business is strongly dependent on the accumulated sales volume, Q, where

$$Q = Q_0 + \int_0^t V\,dt \tag{4}$$

i.e.,

$$V = dQ/dt$$

where Q_0 is the initial value of the accumulated volume. These examples suggest that a reasonable way to add dynamics to conventional price theory is to recognize the fact that the sales volume and the unit cost are functions of the accumulated volume, Q. There will often be dynamic considerations, such as the effect of inflation on material and labor costs, for which a manager does not have a closed model. These can normally be added to the model by introducing an explicit dependence on time, t. We conclude that equations (1) and (2) should be replaced by

$$V = V(P, Q, t) \text{ and} \tag{5}$$

$$C = C(V, Q, t). \tag{6}$$

Note that the prescription for optimizing instantaneous profit flow, (3), still holds.

But now we can turn to a much more appropriate parameter for long run planning; namely, the discounted integral of profits obtained throughout the life of the product or some other planning period, π_D, where

$$\pi_D = \int_0^{t_f} [P - C]V \exp[-\delta t]\, dt, \tag{7}$$

t_f is the time span of our planning period, and δ is the annual rate for discounting future profits.

As we shall see below, if we are given specific expressions for the relations implied by equations (5) and (6), a specific discount rate, δ, and the initial values of the accumulated volume, Q_0, and the unit cost, C_0, our dynamic price model is completely defined. Given a price strategy which relates the price to any of the other variables, we can solve for the accumulated volume, Q, and the unit cost, C, as functions of time. The results can be substituted into equation (7) to calculate the discounted accumulated profit. In this way, the long run implications of various pricing strategies can be calculated and a comparative judgment can be made within the context of any physical or intuitive constraints that the manager may wish to impose; e.g., a limit on the rate of expansion of production capacity or a minimum acceptable value of instantaneous profit flow. On the other hand, one can approach the problem with no predetermined pricing rule and ask the model to determine that price scenario, $P(t)$, which will result in the maximum long run accumulated profit. This can be done by substituting equations (5) and (6) into (7) and applying standard optimization techniques to find that scenario, $P(t)$, which maximizes the integral. Using accepted numerical techniques, [2] and [6], we can optimize the accumulated profit, π_D, subject once again, to any constraints which a manager wishes to impose.

4. The Evolution of Costs

In this section, we shall review the experience curve phenomenon which illustrates the kind of functional dependence that might be used in (6).

The learning curve phenomenon has been applied for many years to project labor costs, [4]. The Boston Consulting Group has noted that in many industries this phenomenon can be generalized to include total unit cost. They have observed that the total unit cost (in constant dollars) declines by 20% to 30% every time the accumulated volume is doubled, [8]. Mathematically this is equivalent to saying that

$$C = C_0[Q_0/Q]^\alpha \tag{8}$$

Where C is the unit cost, Q is the accumulated sales volume, C_0 is the initial unit cost, Q_0 is the initial accumulated volume and α is a constant which falls in the range

$$0.3 \leqq \alpha \leqq 0.5.$$

This phenomenon represents a strong evolutionary force for a new growth business.

The open literature discussion of the experience curve has considered the total unit cost as a evolving aggregate. Some industrial practitioners have found it convenient to assign different experience curves to the various cost components; e.g., labor, materials, overhead, promotion, etc. This approach can be generalized further by taking the projected effects of inflation into account; i.e., multiplying the right hand side of (8), which is in constant dollars, by a time dependent factor which allows for the managers anticipation of the effect of inflation on his labor, materials and other costs.

5. The Evolution of Demand

A number of mathematical models have been used to examine the spread of innovations, including new products, through a population. In order to illustrate the kind of relation one should use in equation (5), we shall consider a specific, relatively simple model. It is an epidemic model which has been used, in retrospect, by Bass to explain the penetration of many consumer durables in the American market, [1]. Bass's growth model applies to initial purchases only. It must eventually be supplemented with a replacement market model. The replacement purchases will come into the picture as the new product reaches a mature stage. For the purpose of our discussion, we will assume that management is only interested in planning for the initial sales into a market with a maximum sales potential of Q_M. If, for example, we were selling refrigerators, Q_M would be the total number of households based on the assumption that each household will be a likely candidate to buy one refrigerator. The size of the initial purchase market Q_M is basically a judgmental input.

The Bass model assumes that there are essentially two basic kinds of purchasers. The first group, innovators, make an independent decision to buy or not to buy. The volume of sales to this group is simply proportional to the number of potential customers who do not already own the product, $Q_M - Q$. The second group consists of people who are sensitive to the actions of their peers. Sales to the second group is again proportional to the number of people who do not have the product, $Q_M - Q$; but it is also proportional to the number of people who do have the product, Q. Summing these two terms, we obtain the epidemic equation for new product sales.

$$V = \underbrace{\gamma_1[Q_M - Q]}_{\text{innovators}} + \underbrace{\gamma_2[Q_M - Q]Q}_{\text{imitators}} \tag{9}$$

where γ_1 and γ_2 are constants of proportionality. This can be rearranged into the following, more convenient, form

$$V = A[1 - Q/Q_M][a + Q/Q_M] \tag{10}$$

where

$$A = \gamma_2 Q_M^2$$

and

$$a = \gamma_1/\gamma_2 Q_M.$$

Bass's studies indicate that, for consumer durables, the constant a is typically a few hundredths. This implies that innovators are only a dominant factor in the marketplace during the short period required to achieve the first several percent of market penetration. We expect this to be true for many markets. Furthermore, he assumed that the quantity A in (10) is strictly constant. Actually, A should be a function of a number of economic variables such as advertising, promotion and price. In other words, equation (10) is really telling us how our product will penetrate the market with a given price—assuming that other economic variables are held constant; but, this is exactly what we require to determine how the demand schedule evolves in time. If we recognize that the quantity A, in (10), is a function of price, $A = A(P)$, we have arrived at a dynamic model of demand which is a suitable input to our price theory for some business situations. By keeping factors other than price constant, we have arrived at a dynamic analog of the conventional price theory described in §2. Modifications to include competitive gaming, promotion costs, and multiple products are also possible. The purpose of the current paper is to establish the advantages of the dynamic approach. The details must be tailored to each situation. We shall consider an illustrative example in the next section.

6. An Illustrative Example

Consider a product, let us say a semiconductor device, which the company thinks is a new and unique product. The market cannot be saturated instantaneously even at low prices. Some market development is required (e.g., computer manufacturers will not use a new technology until it has demonstrated reliability in other applications). Experience with similar products leads us to expect the magnitude of the elasticity of demand, ϵ, to increase with increasing price; i.e.

$$\epsilon \equiv \frac{P}{V}\frac{\partial V}{\partial P} \propto -P \tag{11}$$

We expect, therefore, to be able to fit the demand curve at any instant in time with a demand function of the form

$$V \propto \exp[-BP] \tag{12}$$

where B is a constant. Preliminary estimates as regards to the demand suggest that during the initial period, when our accumulated volume is much smaller than the ultimate sales potential, our market will be described by the following demand func-

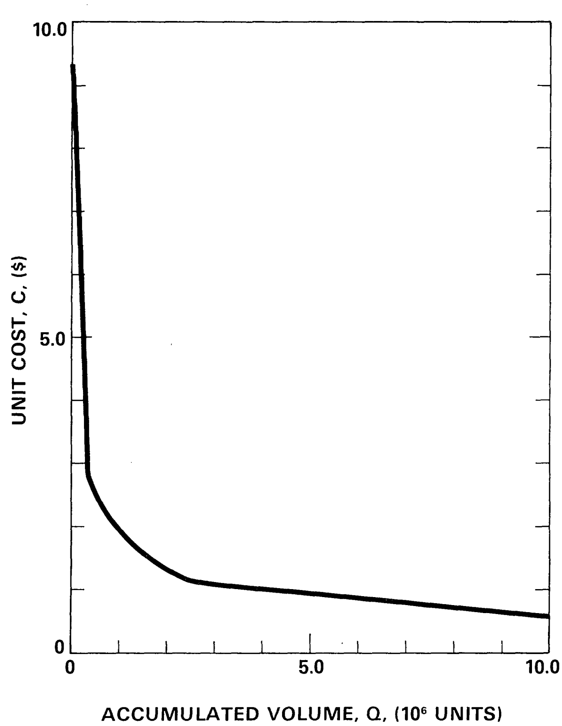

FIGURE 1. Evolution of Demand, (14). The Bass market penetration model is applied to a demand schedule of the form $V = \bar{A}\exp(-BP)$ to illustrate how accumulated activity in the marketplace feeds back on the market itself to create a shift in demand. V = sales volume, P = unit Price, Q = accumulated sales volume.

tion:

$$V = 10^6 \exp[-0.35 P]; \quad \text{when} \quad Q \ll Q_M. \tag{13}$$

Furthermore, based on experiences with similar products, we expect the Bass model to provide an adequate description of the market's evolution with innovators dominating the market until a roughly 3% penetration has been achieved. We can combine these assumptions about the demand for our product with the Bass model, as expressed in (10), to obtain a final dynamic model of the market for our product,

$$V = 3.3 \times 10^7[0.03 + Q/Q_M][1 - Q/Q_M] \exp[-0.35 P] \tag{14}$$

where we have used (13) and the other assumptions about our market to make the price dependence of the quantity A explicit. See Figure 1.

We anticipate initial product sales to saturate at $Q_M = 10{,}000{,}000$. For simplicity, we do not wish to take replacement sales into account. Production experience at $t = 0$; i.e., Q_0, from R&D and pilot plant operation is 10,000 units. Pilot plant experience indicates an initial unit cost, C_0, of \$10; but, based on experience with similar products we expect costs to follow an experience curve corresponding to a 25% decline every time the accumulated volume, Q, is doubled. This corresponds approximately to $\alpha = 0.4$ in (8),

$$C = 10(10{,}000/Q)^{0.4} \tag{15}$$

See Figure 2. To keep things simple we have assumed that C is not a function of the

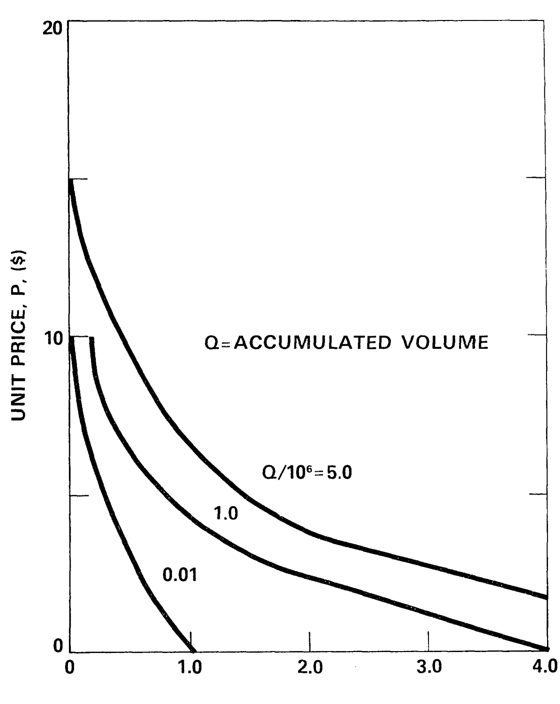

FIGURE 2. Evolution of Costs, (15). The experience curve is used to illustrate how accumulated activity in the marketplace feeds back on production to create a decline in unit cost. C = unit cost, Q = accumulated volume, C_o = initial unit cost, Q_o = initial accumulated volume.

instantaneous value of the sales volume, V; i.e., fixed costs are negligible. Including fixed costs presents no additional difficulties in the ultimate solution of the problem. We wish to plan our price strategy for the next five years, $t_f = 5$. Because of the risk associated with the venture, we wish to discount future profits at an annual rate of 40% to place emphasis on current profits.[1]

Our problem is now completely defined. The ideas outlined in §3 can be used to plan within the context of any pricing rule-of-thumb and to explore its long run consequences. Consider some specific examples.

A) *Marginal Pricing*

The marginal pricer would apply condition (3) to (14) and (15) to obtain

$$P = C + 2.86 \tag{16}$$

as the unit price which optimizes the instantaneous profit flow.[2]

B) *Constant-Return-On-Sales*

If one insists on a constant-return-on-sales, μ, one would choose

$$P = C/[1 - \mu]. \tag{17}$$

C) *Constant Price*

If one wishes to keep the price stable one would choose

$$P = P' = \text{constant}. \tag{18}$$

The evolution of the business under these pricing strategies can be obtained by

[1] Discount rates will typically vary from 10 to 25%. We have chosen an exceptionally high discount rate to demonstrate the fact that even when future profits are strongly discounted, a dynamic price model will provide considerably better long run results than conventional models which optimize current profit flow.

[2] Expanding condition (3), we can write

$$V\frac{\partial P}{\partial V} + P = V\frac{\partial C}{\partial V} + C\,.$$

By differentiating (15), we get

$$\frac{\partial C}{\partial V} = 0.$$

Rearranging (14),

$$\exp\,(.35P) = 3.3 \times 10^7\,[.03 + Q/Qm]\,[1 - Q/Qm]/V.$$

Differentiating with respect to V, we get

$$\frac{\partial P}{\partial V}\,0.35\,\exp\,(.35P) = -\,3.3 \times 10^7\,[.03 + Q/Qm]\,[1 - Q/Qm]/V^2.$$

Simplifying,

$$\frac{\partial P}{\partial V} = -\,\frac{1}{0.35\;V}$$

Substituting above, we obtain

$$-\,\frac{1}{0.35} + P = C\,.$$

TABLE 1

Evolution of the Business Under Various Price Strategies. Year end values of several key parameters are show as the business evolves under four different price strategies: 1) Marginal pricing, equation (13); 2) Optimum constant-return-on-sales, equation (14) with μ = 0.26; 3) Optimum constant-price, equation (15) with P′ = 3.25, and 4) The optimum price scenario. The key parameter for making value judgments is the discounted accumulated profit, π_D, defined in (7). In this illustrative example, future profits were discounted at a rate of 40% per year. Note that prices are changed on a quarterly basis. Unit prices shown are appropriate for last quarter of each year. Costs evolve continuously. Unit costs shown are for the last unit made in each year. Marginal and constant-returns pricing rules are strictly enforced only at the beginning of each quarter.

Price Strategy	Time (Yr.) End	Unit Price ($) ($P$)	Unit Cost ($) ($C$)	Accum. Vol. (10^3 Units) (Q)	Disc. Accum. Profit (10^3 $)	Disc. Accum. Costs (10^3 $)
Marginal	0	12.86	10.00	10	0	
	1	9.13	6.73	32	56	
	2	6.73	3.87	107	181	
	3	5.44	2.58	295	386	
	4	4.68	1.82	703	684	
	5	4.29	1.34	1546	1100	968
Optimum constant return (26%)	0	13.89	10.00	10	0	
	1	9.42	6.78	26	47	
	2	5.17	3.73	118	161	
	3	2.73	1.96	586	379	
	4	1.47	1.06	2730	741	
	5	1.02	.69	8239	1114	2257
Optimum constant price	0	3.25	10.00	10	0	
	1	3.25	2.06	522	−14	
	2	3.25	1.27	1739	1062	
	3	3.25	.90	4066	2917	
	4	3.25	.73	6955	4700	
	5	3.25	.66	8915	5582	4044
Optimum strategy	0	2.82	10.00	10	0	
	1	2.96	1.42	1322	−793	
	2	4.15	1.00	3180	1482	
	3	4.57	.83	5108	3932	
	4	4.42	.73	6883	5652	
	5	3.55	.67	8508	6602	4764

simply substituting, (16), (17), or (18) into (14) which can be solved along with (15) and (7) to yield the resulting time evolution of all of the pertinent variables. In all of our calculations we have assumed that prices are reviewed and changed, if the rule requires it, on a quarterly basis. Pricing rules (17) and (18) can be applied for various values of μ or P' to determine that value which optimizes the discounted accumulated profit within the context of the pricing rule. The resulting evolution of the business is displayed in the first three sections of Table 1.

D) *Optimum Pricing*

Finally, we can approach the problem with no preconception about price and ask what price scenario, $P(t)$, optimizes the discounted accumulated profit, π_D. If all functions were smooth and all constraints equality constraints, we could apply the calculus of variations to equation (7) and obtain the appropriate Euler equation to determine the optimum $P(t)$. In most real cases, however, many functions will be discontinuous; e.g., material costs will be a discontinuous function of, V, due to volume discounts. Furthermore most constraints will be in the form of inequalities; e.g., the production rate, V, must build up at less than some maximum rate. In these cases, it is easier to optimize (7) numerically using dynamic programming, [2] or [4]. Applying Bellman's Principle of Optimality we can work the problem backwards.

We have 20 decision stages, every quarter for five years. Assume some accumulated volume at the end of the planning period, $Q(20)$. Equations (14), (15), and (18) can be used to determine that $P(20)$ and $V(20)$ which optimize the discounted profit obtained in the last quarter, $\Delta\pi_D(20)$. We then have $Q(19) = Q(20) - V(20)/4$. The process can be repeated to obtain that price, $P(19)$, and volume, $V(19)$, for the 19th quarter which optimizes the accumulated profit for the last two quarters, $\Delta\pi_D(19) + \Delta\pi_D(20)$. This process can be repeated until the entire optimum evolution corresponding to a given final accumulated volume, $Q(20)$, has been obtained. The entire procedure can then be repeated for many values of $Q(20)$ until we obtain the optimum scenario corresponding to $Q(0) = 10{,}000$. This is a quick and inexpensive procedure with a modern computer. A typical run would take less than 4 to 5 minutes of CPU time. The result is displayed in the last section of Table 1. This basic procedure can also be used to seek the optimum scenario within the context of any constraint which a manager may wish to impose.

The unit prices displayed in Table 1 are the prices which would prevail, for the various strategies, during the last quarter of each year of the planning period. Recall that unit prices are reviewed and changed on a quarterly basis throughout the year. They are kept constant during any given quarter. Unit costs, on the other hand, are assumed to decline continuously because of the experience curve phenomenon. The unit costs displayed in the table represent the cost of the last unit made in the given year. Since prices are fixed during each quarter and costs continue to evolve, the marginal and the constant-returns rules are only strictly enforced at the beginning of each quarter.

Consider the implications of Table 1 for our specific example. *Even when future profits are heavily discounted, 40%/year in our case, marginal pricing, which optimizes instantaneous profit flow, is far from optimum.* The dynamic optimum scenario leads to a discounted accumulated profit which is 6 times higher. The philosophy of constant-return-on-sales is also not very successful for this product even when the optimum return, $\mu = 0.26$, is chosen. Note that both the marginal and optimum-returns strategies suggest prices in the critical early stages which invite competition. On the other hand, the two most successful strategies, optimum constant-price and the optimum scenario, call for initial prices, \$3.25 and \$2.82, well below the initial cost of \$10 per unit. The results suggests that penetration pricing can be completely justified from the point of view of long run profits. Dynamic considerations clarify the extent to which accumulated volume must feed back on costs, via the experience curve, and the market, via the penetration model, to optimize profits in the long run. In this example the two most successful pricing strategies offer a significant market stability with little sacrifice in long run profit.

Conclusions

The major points established in this paper are: classic marginal pricing is far from optimum for a rapidly evolving business; more appropriate dynamic models can be formulated if one has some feeling for the dominant evolutionary forces in the business environment; and, planning based on the dynamic models can lead to a significant improvement in long run profit performance.

Our discussion has not included the complications of promotion, multiple products or competitive gaming; but, such generalization is certainly possible. In its current form, the model can be thought of as applying to the case of a firm which has a temporary monopoly position in a market because of a technological lead, patents, etc; or, as the appropriate mean behavior for an industry around which an individual businessman should plot his competitive strategy. Our specific example shows that in a market with growth potential, even a monopolist, motivated by nothing but a desire for high profits in the long run, should follow surprisingly aggressive pricing strategies characterized by the absorption of sizable losses during a significant fraction of the initial planning period. Since the dynamic model suggest extremely aggressive pricing for the monopolist, corrections to take competition into account should be small compared to those required in the classic static model.

The major conclusion of our study, however, is that a manager who has some insight into the evolution of his market and of his fixed and variable costs can incorporate these ideas into a dynamic pricing model which can greatly enhance his long run performance.

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