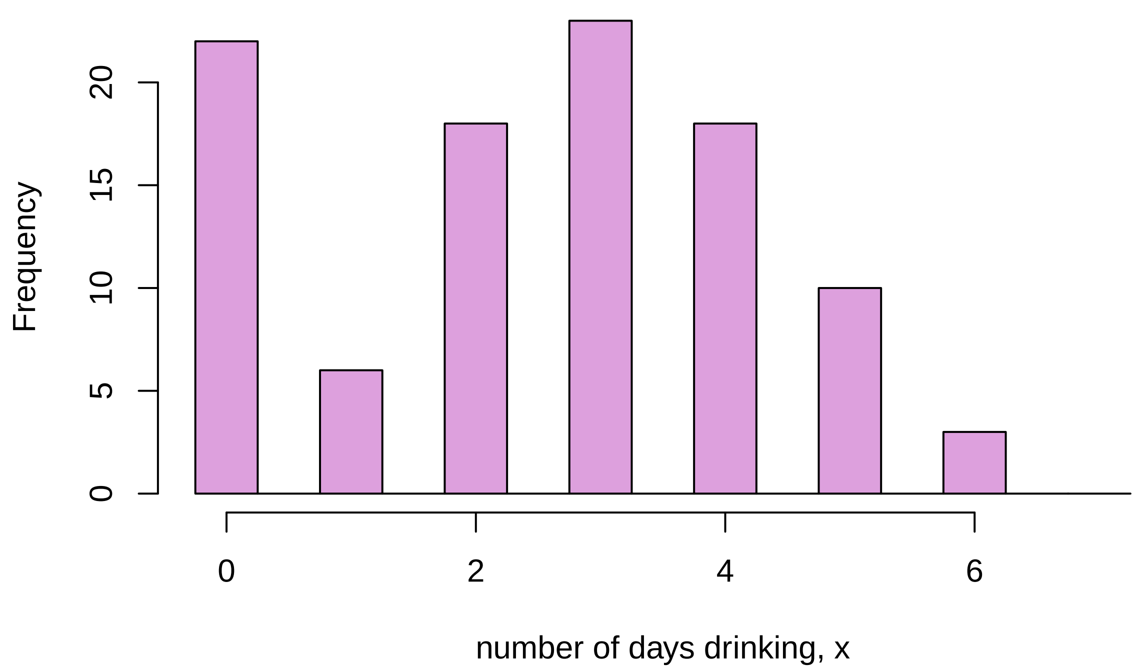
Problem 1.

Let's get back to the drinkers and teetotallers data.

```
x <- rep(0:6,c(22,6,18,23,18,10,3))
hist(x,seq(-.25,7.25,.5),col='plum',xlab='number of days drinking, x',main='')
```



Consider the zero-inflated binomial model. Let $z_i = 1$ if the person i drinks alcohol at least sometimes, and $z_i = 0$ otherwise. And let x_i be the number of days in the past week the person i reported as alcohol drinking days.

$$Pr(x_i = 1|z_i = 0) = 1 - Pr(x_i = 0|z_i = 0) = 0$$

$$x_i|z_i = 1 \sim \text{BIN}(7, p)$$

and the prior for p:

$$p \sim Beta(a, b)$$

for some prior parameters a and b.

Furthermore, assume that z_i has a further Bernoulli distribution with probability ω:

$$z_i \sim \text{Bern}(\omega),$$

and that the probability parameter ω also has a beta prior (with parameters different from those for the prior of p):

$$\omega \sim Beta(a_\omega, b_\omega)$$

(a) Derive the posterior conditional distribution of z_i given ω and x_i.

Hint: z_i can only take two values, so it is enough to evaluate the $Pr(z_i = 1|x_i, \omega, p)$. The solution for $x_i > 0$ should be obvious. So, the only remaining challenge is to find $Pr(z_i = 1|x_i = 0, \omega, p)$. Use Bayes' formula. **4pt**