

**Problem 1:** Suppose we wish to construct a password of length 20, which may include the following characters: 9 special characters, 10 numbers, 26 lowercase letters, and 26 uppercase letters. The password contains exactly 5 distinct special characters, and the first character is NOT a special character. Also, repetition of lowercase letters, numbers, and uppercase letters is allowed. Additionally we have the restrictions:

- The password contains at least one number

AND

- At least two of the five special characters are consecutive

Find the total number of passwords possible with the (simultaneous) restrictions listed above by **using the Principle of Inclusion-Exclusion and defining sets explicitly**.

**Problem 2:** Suppose we want to assign 46 computer science students to work on teams to complete 12 projects. Note that it is possible for some projects to receive no qualified students to work on the project.

We have the following restrictions on the number of students on the teams:

- Project teams 3 and 7 have **exactly** 8 students.
- Project teams 1, 2, and 12 have **no more than** 5 students.
- Project teams 4, 9, 10, and 11 have **at least** 3 students, but **strictly less than** 9 students.
- Project teams 5, 6, and 8 have **at least** 4 students but **strictly less than** 16 students.

Find the total number of ways we can distribute 46 students to the 12 project teams by

- (i) Modeling this combinatorial problem as an integer equation with restrictions
- (ii) Constructing a generating function
- (ii) Computing the relevant coefficient