2) Consider the function $\varphi : \mathbb{R} \times (-1,1) \mapsto \mathbb{R}^3$ given by

$$\varphi(u,v) = \left(\left(1 + v\,sin\left(\frac{u}{2}\right)\right) cos\,u\ , \left(1 + v\,sin\,\left(\frac{u}{2}\right)\right) sin\,u\ , v\,cos\left(\frac{u}{2}\right)\right) \in \mathbb{R}^3$$

for $u \in \mathbb{R}\,, v\ \in\ (-1,1)$. The *Mobius Strip M* is the image of this function, i.e:

$$M = Im\,\varphi$$

a) Show that *M* is a C^∞ two-dimensional submanifold. (Suggestion: restrict φ to obtain parametrizations)

b) Calculate, in (u,v), the vector product

$$Z = \frac{\partial\varphi}{\partial u} \times \frac{\partial\varphi}{\partial v}$$

where $\frac{\partial\varphi}{\partial u} \coloneqq D\varphi(1{,}0)$ and $\frac{\partial\varphi}{\partial v} \coloneqq D\varphi(0{,}1)$.

c) A *vector field* onto a submanifold $N \subset \mathbb{R}^n$ is an application $X : N \to \mathbb{R}^n$. Said field is *tangent* (to *N*) if $X(p) \in T_pN$ for every $p \in N$, and *normal* (to *N*) if $X(p) \in (T_pN)^\perp$ for every $p \in N$. Show that there is no normal, continuous non-zero-in-every-point field $X : M \to \mathbb{R}^3$ onto the Mobius Strip. (Suggestion: note that if there was such an *X*, we should have that $X \circ \varphi(u,v) = f(u,v)Z(u,v)$ where *f* is continuous and never zeroes. Evaluate that in points $(u,0)$ to reach a contradiction.)