Exercise 1

Show that, for any three events E, F, G of a sample space S,

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(E \cap G) - P(F \cap G) + P(E \cap F \cap G)$$

Exercise 2

Let $S = \{1, 2, \ldots, N\}$ the sample space. Assume that all the outcomes are equally likely to occur; i.e., $P(\{1\}) = P(\{2\}) = \cdots = P(\{N\})$. Show that $P(\{i\}) = \frac{1}{N}$ for each $i = 1, 2, \ldots, N$.

Problem 5. Show that if E and F are independent, then E and F^C are also independent.

[Hint: Write $P(E) = P(E \cap F) + P(E \cap F^C)$, then use the independence of E and F (i.e., $P(E \cap F) = P(E)P(F)$).]

Problem 6. Show that if E, F, G are independent events, then E is independent of $F \cap G$ and $F \cup G$.

Exercise 12

Let M be an $n \times n$ matrix. Show that if $\vec{v} = V$ is a solution to the equation

$$M\vec{v} = \vec{v}$$

then for any real number c, $\vec{v} = cV$ is also a solution to this equation.