

Miner

The miner can reach the exit in 0 days (picks exist immediately), 2 days (picks the door of two days, then escapes), 3 days (picks the door of three days, then escapes) or 5 days (picks the 2 day door, then 3 day, then escapes, or first 3 day, 2 day, escape).

The expected number to days to escape is:

$$E(\text{Days}) = 0 \cdot P(0 \text{ days}) + 2 \cdot P(2 \text{ days}) + 3 \cdot P(3 \text{ days}) + 5 \cdot P(5 \text{ days})$$

$$P(2 \text{ days}) = P(\text{wrong door of 2 days})P(\text{escape}|\text{wrong door of 2 days}) = \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)$$

$$P(3 \text{ days}) = P(\text{wrong door of 3 days})P(\text{escape}|\text{wrong door of 3 days}) = \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)$$

$$P(5 \text{ days})$$

$$\begin{aligned} &= P(\text{wrong door of 2 days})P(\text{wrong door of 3 days}|\text{wrong door of 2 days})P(\text{escape}|\text{all wrong doors chosen}) \\ &+ P(\text{wrong door of 3 days})P(\text{wrong door of 2 days}|\text{wrong door of 3 days})P(\text{escape}|\text{all wrong doors chosen}) \\ &= \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)(1) + \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)(1) \end{aligned}$$

$$E(\text{Days}) = 2 \left(\frac{1}{3}\right)\left(\frac{1}{2}\right) + 3 \left(\frac{1}{3}\right)\left(\frac{1}{2}\right) + 5 \left[2 \left(\frac{1}{3}\right)\left(\frac{1}{2}\right) \right] = \frac{1}{3} + \frac{1}{2} + \frac{5}{3} = 2.5 \text{ days}$$

The variance of the number to days to escape is:

$$\begin{aligned} V(\text{Days}) &= E(\text{Days}^2) - E(\text{Days})^2 = 0 \cdot P(0 \text{ days}) + 4 \cdot P(2 \text{ days}) + 9 \cdot P(3 \text{ days}) + 25 \cdot P(5 \text{ days}) - 2.5^2 \\ &= 4 \left(\frac{1}{3}\right)\left(\frac{1}{2}\right) + 9 \left(\frac{1}{3}\right)\left(\frac{1}{2}\right) + 25 \left[2 \left(\frac{1}{3}\right)\left(\frac{1}{2}\right) \right] - (2.5 \text{ days})^2 = \frac{2}{3} + \frac{3}{2} + \frac{25}{3} - 6.25 = 4.25 \text{ days}^2 \end{aligned}$$