Question: Use induction to prove that

$$1 + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}} \leq 2\sqrt{n} - 1,$$

for all $n \geq 1$.

Base Case: Observe that $1 \leq 1 = 2\sqrt{1} - 1$, and thus the statement holds for $n = 1$.

Inductive Step: Suppose that

$$1 + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{k}} \leq 2\sqrt{k} - 1,$$

for some given $k \geq 1$. Thus, by the induction hypothesis

$$1 + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} \leq (2\sqrt{k} - 1) + \frac{1}{\sqrt{k+1}}.$$

Therefore, it suffices to demonstrate that $(2\sqrt{k} - 1) + \frac{1}{\sqrt{k+1}} \leq 2\sqrt{k+1} - 1$.
It's certainly the case that $k^2 + k \leq k^2 + k + \frac{1}{4}$ for all $k \geq 1$, and thus, by factoring each side

$$k(k+1) \leq \left(k + \frac{1}{2}\right)^2 \tag{1}$$

$$\sqrt{k(k+1)} \leq k + \frac{1}{2} \tag{2}$$

$$\sqrt{k(k+1)} + \frac{1}{2} \leq k + 1 \tag{3}$$

$$2\sqrt{k(k+1)} + 1 \leq 2(k+1) \tag{4}$$

$$2\sqrt{k(k+1)} - \sqrt{k+1} + 1 \leq 2(k+1) - \sqrt{k+1}, \tag{5}$$

and finally since $\sqrt{k+1} > 0$ we can divide through to obtain $2\sqrt{k} - 1 + \frac{1}{\sqrt{k+1}} \leq 2\sqrt{k+1} - 1$ as desired.

Thus we have shown by the principal of mathematical induction, that

$$1 + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}} \leq 2\sqrt{n} - 1,$$

for all $n \geq 1$.